4/23/15

1. $\int \sin(x)^5 \cos(x)\, dx$

$\boxed{u = \sin(x) \qquad du = \cos(x)\,dx}$

$= \int u^5\, du = \frac{1}{6} u^6 + C = \boxed{\frac{1}{6}\sin(x)^6 + C}$

2. $\int \frac{2\,dx}{x+3} = 2\int \frac{dx}{x+3} = 2\int \frac{du}{u} = 2\ln(u) + C$

$\boxed{u = x+3 \qquad du = dx}$ $= \boxed{2\ln(x+3) + C}$

3. $\int 1 + x + x^2 + x^3\, dx = \boxed{x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + C}$

4. $\int \frac{x+1}{x}\,dx = \int 1 + \frac{1}{x}\,dx = \boxed{x + \ln(x) + C}$

5. $\int \frac{dx}{x^2+9} = \frac{1}{9}\int \frac{dx}{\left(\frac{x}{3}\right)^2+1} = \frac{1}{9}\int \frac{3\,du}{u^2+1}$

$\boxed{u = \frac{x}{3} \qquad du = \frac{1}{3}dx \Rightarrow dx = 3\,du}$

$= \frac{1}{3}\int \frac{du}{u^2+1} = \frac{1}{3}\arctan(u) + C = \boxed{\frac{1}{3}\arctan\left(\frac{x}{3}\right) + C}$

6. $\displaystyle\int_{-3}^{3} \ln(x^2+1)\sin(x) + 1 \quad dx$

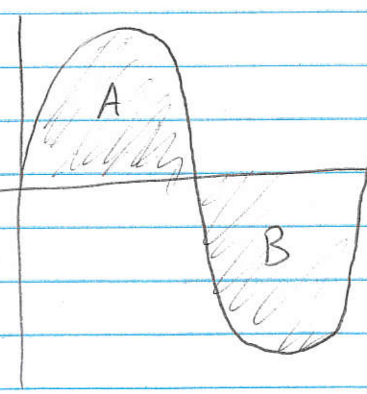
$$= \int_{-3}^{3} \ln(x^2+1)\sin(x)\,dx + \int_{-3}^{3} dx$$

(Since $\ln(x^2+1)\sin(x)$ is an odd function, the integral on the left vanishes.)

$$= 0 + \int_{-3}^{3} dx = \Big[x\Big]_{-3}^{3} = 3-(-3) = \boxed{6}$$

7. $\displaystyle\int_0^{2\pi} \sin(x)^{17}\,dx = \boxed{0}$

Bump (A) cancels with bump (B).



8. $\displaystyle\frac{d}{ds}\int_s^{1-s} e^{e^x}\,dx$

Let $F(x)$ be an antiderivative of e^{e^x}, so $F'(x) = e^{e^x}$. Then $\displaystyle\int_s^{1-s} e^{e^x}dx = F(1-s)-F(s)$.

So $\displaystyle\frac{d}{ds}\int_s^{1-s} e^{e^x}\,dx = \frac{d}{ds}\Big[F(1-s)-F(s)\Big] = F'(1-s)\cdot(-1) - F'(s)$

$$= \boxed{-e^{e^{1-s}} - e^{e^{s}}}$$